Next-to-leading gluonic Reggeons in the high-energy effective action

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We study next-to-leading gluon exchange in high-energy scattering that contributes to the amplitude to order s^0 up to logarithmic corrections. Similar to the leading gluon exchange these contributions can be described in terms of Reggeon exchanges. There are several gluonic Reggeons at the next-to-leading level. Some of them transfer parity or gauge group representations different from the leading gluonic Reggeon. Unlike the leading one they are sensitive to the helicity and transverse momenta of the scattering partons. We extend the high-energy effective action and derive from the action of gluodynamics the terms describing the next-to-leading Reggeons and their interaction in the multi-Regge approximation. [S0556-2821(98)01213-2]

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I. INTRODUCTION

The leading contribution to the high-energy asymptotics of scattering processes in QCD can be described by a Reggeon and its interaction. The exchange of two such leading Reggeized gluons with interactions summed up in the leading $\ln s$ approximation results in the Balitskiĭ-Fadin-Kuraev-Lipatov (BFKL) Pomeron [1], which has been by now successfully applied in the phenomenological analysis of semi-hard processes and in particular of deep-inelastic scattering at small values of the Bjorken variable x. The systematic improvement of the leading $\ln s$ approximation can be organized using the Reggeized gluon concept: The exchange of an even number of leading Reggeized gluons with interactions taken in the same approximation as in the BFKL Pomeron gives rise to a unitarity correction to the latter. The exchange of three Reggeized gluons leads to the odderon [2], the actual role of which in phenomenology is still unclear. Unitarity corrections to the odderon result from the exchange of an odd number of Reggeized gluons. Further next-to-leading $\ln s$ corrections result in corrections to the scattering and production vertices and in new vertices. These corrections are related to going beyond multi-Regge approximation (to be explained below, Sec. II A).

Much effort has been applied in the last years to calculate the latter corrections [3] and also to calculate the Regge singularity induced by multiple exchange of leading Reggeized gluons [4] as well as by higher Reggeon interaction vertices [5]. The Reggeon concept is a starting point of the multiparticle unitarity approach to high-energy scattering [6].

The paper is devoted to the study of the contribution from gluon exchange suppressed by one power of the c.m. system (c.m.s.) energy squared s compared to the leading gluon exchange. We propose to apply the Reggeon concept also to the nonleading exchanges.

Analyzing the high-energy asymptotics it is convenient to consider the Mellin transform of the amplitude with respect to the energy squared s. It is essentially the t-channel partial wave and the Mellin variable j is the complex angular momentum. The leading gluon exchange induces Regge singularities near $j=1$, the nonleading gluon exchanges studied here appear as Reggeons with poles near $j=0$.

There are observables in high-energy scattering to which the nonleading exchanges contribute not just as a small correction. Nonvacuum quantum numbers such as odd (C or P) parity can be transferred by gluons.

Consider the scattering with small momentum transfer of a high-momentum gluon or quark on a source of color fields. The leading interaction contributes to the amplitude proportional to the first power of the large momentum. It conserves helicity of the high-momentum gluon or quark and it is not sensitive to the details of the color source such as its distribution in the transverse (impact parameter) plane or to its spin structure. However, such details are resolved by interactions suppressed by one power of the large momentum compared to the leading one.

The exchange of one leading and one nonleading gluonic Reggeon gives a contribution to the small-x asymptotics of the spin structure function $g_1(x)$ of the proton [7] measuring the helicity asymmetry. The exchange of two nonleading gluonic Reggeons contributes to the small-x asymptotics of the spin structure function $F_3^\gamma(x)$ of the photon (spin-1 target), measuring the transverse polarization asymmetry of gluons (gluon transversity) [8].

The high-energy effective action provides a technical framework for formulating and analyzing the problems of high-energy scattering in gauge theories. It has been proposed originally as a summary of the leading $\ln s$ results for the leading gluon exchange in a simple form and a starting point for going beyond this approximation [9]. Then it has been understood that it is indeed an effective action in the sense of Wilson. It can be derived from the original action by separating the fields into modes and integrating over those modes that do not correspond neither to scattering quanta (partons) nor to exchanged quanta [10]. The effective action has been studied up to now in the multi-Regge approximation for describing the leading gluon and leading fermion exchanges. There are results going beyond this approximation [11].

Here we are going to extend the procedure to nonleading gluon exchanges contributing to $\mathcal{O}(s^0)$ to the amplitude. We restrict ourselves for simplicity to the case without fermions, i.e., to pure gluodynamics. We stay within the multi-Regge approximation improving the known procedure by keeping terms suppressed by one power of s. The experience from the exercise in high-energy scattering in (linearized) gravity [12] helped us to optimize the extensive calculations.

The structure of the effective vertices with fermions included can be obtained from the gluonic vertices by supersymmetry transformations relying on the similarity of QCD to supersymmetric Yang-Mills theory. A short description of our results including a discussion about fermions has been published earlier [13].

Some of the technical steps in our procedure can be justified only in the framework of perturbation theory, the applicability of which is restricted to the semihard region. There we have besides the energy squared a second large momentum scale (momentum transfer or virtuality Q^2) which is much smaller than s but still large compared to the hadronic scale. Referring to dominating momentum configurations in this perturbative Regge region, we can give the inverse derivatives appearing in the calculations and in the final result a meaning, since the typical longitudinal momenta are not small and small transverse momenta should not be essential either. Nonlocal interactions are an essential feature of our effective action.

In the next section we discuss the separation of modes according to the multi-Regge kinematics. This is the first essential step towards the effective action. We start from the Yang-Mills action in the lightlike axial gauge with the redundant field components eliminated. In this way we have a direct correspondence between the fields and the physical degrees of freedom. This gauge is convenient but can be avoided. The resulting action does not depend on gauge. We study the impact of the separation of modes on the triple and quartic interaction terms. Thinking of the physical situations of high-energy gluon scattering in an external field and of gluon-gluon quasielastic scattering gives us a guideline to collect the most important terms for deriving the vertices of the effective action. We study the quartic terms contributing to high-energy elastic scattering. The ones corresponding to s-channel gluon exchange far off-shell are calculated in Sec. III. Eliminating these ''heavy modes'' is the second essential step towards the effective action.

The resulting terms describing effectively quasielastic scattering are analyzed in Sec. IV. We obtain a sum of terms of the form current times current, where the currents describe the gluons scattering with relatively small momentum transfer. This factorizability is essential for identifying the Regge exchanges. At the end of Sec. IV we write down the effective action of quasielastic scattering introducing pairs of pre-Reggeon fields.

It becomes clear that due to parity symmetry, interchanging the incoming particles, only a part of quartic terms is sufficient to obtain the effective action for quasielastic scattering. This observation is used to cut short the calculations in Sec. V where we study the terms of order 5 describing effectively the inelastic $2\to 3$ process in the multi-Regge kinematics. By factorization we obtain the effective production vertices. The high-energy effective action is obtained by adding the production vertices to the effective action of quasielastic scattering. The features of this action are discussed in the last section.

II. SEPARATION OF MODES

A. Multi-Regge kinematics

It is convenient to start from the Yang-Mills action in the light-cone axial gauge $A_-=0$:

$$\mathcal{L}=\mathcal{L}^{(2)}+\mathcal{L}^{(3)}+\mathcal{L}^{(4)},$$

$$\mathcal{L}^{(2)}=-2A^{a*}(\partial_+\partial_- - \partial\partial^*)A^a,$$

$$\mathcal{L}^{(3)}=-\frac{g}{2}\,J_-^a\mathcal{A}_+^a-\frac{g}{2}\,j^a\mathcal{A}'^a,$$

$$\mathcal{L}^{(4)}=\frac{g^2}{8}\,J_-^a\partial_-^{-2}J_-^a-\frac{g^2}{8}\,j^aj^a. \tag{2.1}$$

We use light-cone components for the longitudinal part of vectors and complex numbers for the transverse part [10]. The space-time derivatives are normalized such that $\partial_+x_- =\partial_-x_+=\partial x=\partial^*x^*=1$. The gluon field is represented by the transverse gauge potential A^a, A^{a*}. It enters the interaction terms (2.1) in the combinations

$$\mathcal{A}_+^a=\partial_-^{-1}(\partial A^a+\partial^*A^{a*}),\quad \mathcal{A}'^a=i(\partial A^a-\partial^*A^{a*}) \tag{2.2}$$

and in the currents

$$J_-^a=i(A^*T^a\overset{\leftrightarrow}{\partial}_-A),\quad j^a=(A^*T^aA). \tag{2.3}$$

In the following we encounter besides the longitudinal components J_-^a also the transverse components J^a,J^{a*} of the vector current (obtained by replacing ∂_- by ∂^* and ∂, respectively). We use the abbreviation

$$(AT^aB)=-if^{abc}A^bB^c \tag{2.4}$$

with f^{abc} the structure constants of SU(N).

The notations are chosen such that there is a close relation to the leading terms of the effective action [10]: The expression $\mathcal{A}_+^a$ (2.2) describes the leading gluonic Reggeon and the current J_-^a determines the leading scattering vertex. We shall see that some of the nonleading gluonic Reggeons are described by the expression $\mathcal{A}'^a$ (2.2) in terms of the original gluon field A^a. From the point of view of momentum representation $\mathcal{A}_+^a$ and $\mathcal{A}'^a$ represent the projections of the transverse gauge potential $A^a(k)$, the first parallel to the transverse part of its momentum κ^μ and the second orthogonal to κ^μ. The nonleading scattering vertices involve besides the

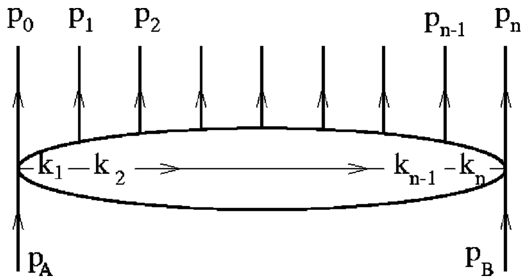

FIG. 1. The inelastic process in the multi-Regge kinematics.

current j^a other currents such as J^a, J^{a*}. Removing the redundant field components in the light-cone gauge is convenient because now the complex field A^a is directly related to the gluonic degrees of freedom and as we have stated already in Sec. I introducing this gauge is a technical step which can be avoided since the effective action is gauge invariant.

We separate the field into modes $A = A_t + A_s + A_1$. A_t are the momentum modes typical for exchanged gluons, A_s are the modes typical for scattering gluons, and A_1 are the heavy modes, which do not contribute directly to the scattering or exchange and will be integrated out.

The modes are separated according to the multi-Regge kinematics, i.e., the momentum configuration of a multiparticle s-channel (intermediate) state (p_l, $l = 0,1, \ldots, n$) giving the dominant contribution in the leading $\ln s$ approximation, see Fig. 1. Decomposing the transferred momenta $k_l = p_A - \Sigma_{i=0}^{l-1} p_i$ with respect to the (almost lightlike) momenta of incoming particles p_A, p_B,

$$k^\mu = \sqrt{\frac{1}{s}}\,(k_+ p_B^\mu + k_- p_A^\mu) + \kappa^\mu, \tag{2.5}$$

the multi-Regge kinematics is characterized by the conditions

$$|k_{+n}| \gg \ldots \gg |k_{+1}|, \quad |k_{-n}| \ll \ldots \ll |k_{-1}|,$$
$$|k_{+l} k_{-l}| \ll |\kappa_l|^2, \quad s_l = |k_{-l-1} k_{+l+1}| \gg |\kappa_l|^2, \tag{2.6}$$
$$\prod_{l=1}^{n} s_l = s \prod_{l=2}^{n} |\kappa_l - \kappa_{l-1}|^2.$$

Here κ denotes the transverse (with respect to p_A, p_B) part of the momentum k. It is represented by a four-vector in Eq. (2.5) and in the following it will be represented by a complex number keeping the same notation. The longitudinal momenta are strongly ordered. The subenergies s_l are large compared to the transferred momenta. The longitudinal contribution to the transferred momenta squared is small. In loops the main contribution from s-channel intermediate particles arises from the vicinity of the mass shell. Therefore the modes A_t, A_s, A_1 are characterized by the following conditions:

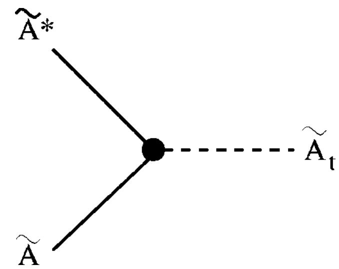

FIG. 2. The graphical illustration of vertices entering Eqs. (2.10) and (2.12).

$$A_t : |k_- k_+| \ll |\kappa|^2,$$
$$A_s : ||k_- k_+| - |\kappa|^2| \ll |\kappa|^2, \tag{2.7}$$
$$A_1 : |k_- k_+| \gg |\kappa|^2.$$

We introduce the mode separation into the action (2.1) by substituting A by $A_s + A_t + A_1$. The kinetic term decomposes into three, one for each of the modes, which follows immediately from momentum conservation:

$$\begin{aligned}\mathcal{L}_{\rm kin}^{(0)} = & -2A_s^{*a}(\partial_+\partial_- - \partial\partial^*)A_s^a \\ & + 2A_t^{*a}\partial\partial^*\left(1 - \frac{\partial_+\partial_-}{\partial\partial^*}\right)A_t^a \\ & - 2A_1^{*a}\partial_+\partial_-\left(1 - \frac{\partial\partial^*}{\partial_+\partial_-}\right)A_1^a.\end{aligned} \tag{2.8}$$

In the kinetic term for A_t and A_1 the second operator in the brackets will be treated as a small one. In the calculations we have to keep the first order in these corrections.

B. The triple interaction

Consider now the triple terms $\mathcal{L}^{(3)}$ of the action (2.1). We introduce the mode decomposition in $\mathcal{L}^{(3)}$ and obtain many terms. The most important terms for our discussion are those, where two of the fields have longitudinal momenta of the same order and third one carries much larger or much smaller longitudinal momentum. We denote by $\mathcal{L}_1^{(3)}$ those terms with one of the three fields in the modes $A_t + A_s = \tilde{A}_t$ and two in the modes $A_s + A_1 = \tilde{A}$:

$$\begin{aligned}\mathcal{L}_1^{(3)} = & -\frac{g}{2}\{i(\tilde{A}^* T^a \overleftrightarrow{\partial}_- \tilde{A})\tilde{\mathcal{A}}_{+t}^a + (\tilde{A}^* T^a \tilde{A})\tilde{\mathcal{A}}_t^{\prime a} \\ & + i(\tilde{A}_t^* T^a \overleftrightarrow{\partial}_- \tilde{A})\tilde{\mathcal{A}}_+^a + i(\tilde{A}^* T^a \overleftrightarrow{\partial}_- \tilde{A}_t)\tilde{\mathcal{A}}_+^a \\ & + (\tilde{A}_t^* T^a \tilde{A})\tilde{\mathcal{A}}^{\prime a} + (\tilde{A}^* T^a \tilde{A}_t)\tilde{\mathcal{A}}^{\prime a}\}.\end{aligned} \tag{2.9}$$

$\tilde{\mathcal{A}}_{+t}$, $\tilde{\mathcal{A}}_t^\prime$ and $\tilde{\mathcal{A}}^\prime, \tilde{\mathcal{A}}_+$ are given by the expressions (2.2) with the fields restricted to the modes $A_t + A_s = \tilde{A}_t$ and $A_s + A_t = \tilde{A}$, respectively. We rearrange the terms in Eq. (2.9) by using the definition (2.4) for the bracket $(A^* T^a A)$ and by performing integration by part in order to put the fields with the mode $\tilde{\mathcal{A}}_t^a$ as the last factor in each term (Fig. 2):

$$\mathcal{L}_1^{(3)} = -\frac{g}{2}\{i(\tilde{A}^* T^a \overleftrightarrow{\partial}_- \tilde{A})\tilde{\mathcal{A}}^a_{+t} + (\tilde{A}^* T^a \tilde{A})\tilde{\mathcal{A}}_t'^{\,a}$$
$$- i(\tilde{A}^* T^a \overleftrightarrow{\partial}^* \tilde{A})\tilde{A}_t^{*a} - i(\tilde{A}^* T^a \overleftrightarrow{\partial} \tilde{A})\tilde{A}_t^a$$
$$+ i(\tilde{A}^* T^a \tilde{A})(\partial \tilde{A}_t^a - \partial^* \tilde{A}_t^{*a}) + 2i(\tilde{A}_+ T^a \tilde{A})\partial_- \tilde{A}_t^{*a}$$
$$- 2i(\tilde{A}^* T^a \tilde{\mathcal{A}}_+)\partial_- \tilde{A}_t^a\}. \tag{2.10}$$

We decompose the fields in the $\tilde{A}_t$ mode in each term into the expressions $\mathcal{A}_+^a$ and $\mathcal{A}'^{\,a}$ (2.2), $\partial \tilde{A}_t = \frac{1}{2}(\partial_- \tilde{A}_{+t} - i\tilde{A}_t')$. Writing this field always as the last factor allows us in the following to omit the subscript $(t,s,1)$ and the tilde referring to the range of modes.

We use the currents J_-^a, J^a, J^{*a}, j^a introduced above in Eq. (2.3) and furthermore

$$J_s^a = \left(\frac{\partial}{\partial_-} A T^a A\right), \quad J_2^a = \left(\frac{\partial^*}{\partial_-} A^* T^a A\right) \tag{2.11}$$

to express the two fields in the $\tilde{A}$ mode in each term and obtain

$$\mathcal{L}_1^{(3)} = -\frac{g}{2}\left(J_-^a - \frac{1}{2}\frac{\partial_-}{\partial\partial^*}(\partial J^a + \partial^* J^{*a}) - i\frac{\partial^2}{\partial\partial^*}[\partial(J_s^a + J_2^a) - \partial^*(J_s^{*a} + J_2^{*a})]\right)\mathcal{A}_+^a$$
$$- \frac{g}{2}\left(2j^a + \frac{i}{2}\frac{1}{\partial\partial^*}(\partial J^a - \partial^* J^{*a}) - \frac{\partial_-}{\partial\partial^*}[\partial(J_s^a + J_2^a) + \partial^*(J_s^{*a} + J_2^{*a})]\right)\mathcal{A}'^{\,a}. \tag{2.12}$$

The separated triple terms (2.12) describe in particular the interaction of a high-energy gluon (modes A_s involved in the currents) with an external field (described by the A_t modes in $\mathcal{A}_+$ and in $\mathcal{A}'$). In the case of scattering with a large momentum component k_- the term with J_-^a gives the leading contribution of order $\mathcal{O}(k_-)$, the terms with J^a, J^{*a}, and j^a contribute to order $\mathcal{O}(k_-^0)$ and the other terms with J_s^a, J_2^a result in corrections of the order $\mathcal{O}(k_-^{-1})$. In describing scattering with large k_+ the ordering goes in the reverse direction. Notice, that the J_s^a terms contribute to helicity flip whereas all other vertices conserve the helicity of the scattering gluon.

C. Quartic interactions and elastic scattering

We introduce the mode separation (2.7) into the quartic term $\mathcal{L}^{(4)}$ of the action (2.1). We pick up the terms with two fields in the modes $\tilde{A} = A_s + A_t$ and the other two in all modes A with the additional condition that the first two have a large longitudinal momentum k_- and the latter two have small k_- but large k_+:

$$\mathcal{L}_{\text{scatt}}^{(4)} = \frac{g^2}{4}\tilde{J}_-^a \frac{1}{\partial_-^2} J_-^a - \frac{g^2}{4}\tilde{J}^a j^a$$
$$- \frac{g^2}{8}\left\{(\tilde{A}^* T^a \overleftrightarrow{\partial}_- A)\frac{1}{\partial_-^2}(\tilde{A}^* T^a \overleftrightarrow{\partial}_- A) + \cdots\right\}$$
$$- \frac{g^2}{8}\{(\tilde{A}^* T^a A)(\tilde{A}^* T^a A) + \cdots\}. \tag{2.13}$$

The ellipsis stands for the three terms obtained from the explicit ones by shifting the tilde signs to the other fields in each of the brackets. Now we look at the derivatives acting on the fields $\tilde{A}$ carrying large k_- modes. We apply approximations such as

$$\frac{1}{\partial_-}(\partial_- \tilde{A}^* T^a A) = (\tilde{A}^* T^a A) - \left(\frac{1}{\partial_-}\tilde{A}^* T^a \partial_- A\right) + \cdots \tag{2.14}$$

and keep only terms which, after changing to momentum representation, are of the order k_-^1 or k_-^0, and obtain

$$\mathcal{L}_{\text{scatt}}^{(4)} = \frac{g^2}{4}\tilde{J}_-^a \frac{1}{\partial_-^2} J_-^a - \frac{g^2}{4}\tilde{J}^a j^a$$
$$- \frac{g^2}{2}(\tilde{A}^* T^a A)(A^* T^a \tilde{A}) + \mathcal{O}(k_-^{-1}). \tag{2.15}$$

We would also like to write the third term as a product of a factor involving $\tilde{A}$ only and a second factor involving A. We use the relation for the generators T^a of the adjoint representation of SU(N),

$$(T^e)_{ab}(T^e)_{cd} - (T^e)_{ac}(T^e)_{bd} = (T^e)_{ad}(T^e)_{cb}, \tag{2.16}$$

and introduce the generators D^r of the reducible representation arising as the symmetric part in the tensor product of the two adjoint representations of SU(N) in order to write

$$(T^e)_{ab}(T^e)_{cd} + (T^e)_{ac}(T^e)_{bd} = (D^r)_{ad}(D^r)_{cb}. \tag{2.17}$$

We introduce the current

$$j_D^r = (A^* D^r A). \tag{2.18}$$

Using relations (2.16), (2.17) and definition (2.18) we obtain

$$\mathcal{L}_{\text{scatt}}^{(4)} = \frac{g^2}{4}\tilde{J}_-^a \frac{1}{\partial_-^2} J_-^a - \frac{g^2}{2}\tilde{J}^a j^a - \frac{g^2}{4}\tilde{J}_D^r j_D^r. \tag{2.19}$$

The separated quartic terms (2.19) contribute to the quasielastic scattering of gluons at high-energy and limited momentum transfer.

The triple terms induce further contributions to quasielastic scattering: Two vertices from $\mathcal{L}_1^{(3)}$ (2.12) can be contracted by t-channel (A_t) or by s-channel (A_1) exchanges.

We consider first the contribution of t-channel exchange. The contribution from heavy modes A_1 will be discussed in the next section.

In one of the vertices the fields in the currents describe scattering gluons with large k_-. We are going to describe high-energy scattering in the accuracy including terms $\mathcal{O}(s^0)$, therefore in this vertex we can disregard the contribution of the currents J_s and J_2 in Eq. (2.12). We restrict the fields in the currents to the modes A_s which are close to mass shell. Then, up to terms $\mathcal{O}(k_-^{-1})$, we can substitute $\partial J + \partial^* J^*$ by $\partial_+ J_-$:

$$\mathcal{L}_1^{(3+)} = -\frac{g}{2}\left(J_-^a - \frac{1}{2}\,\frac{\partial_+\partial_-}{\partial\partial^*}\,J_-^a\right)\mathcal{A}_+^a - \frac{g}{2}\left(2j^a + \frac{i}{2}\,\frac{1}{\partial\partial^*}\,(\partial J^a - \partial^* J^{*a})\right)\mathcal{A}'^a. \tag{2.20}$$

These approximations do not apply to the other vertex the currents of which describe gluons with small k_-. We transform the currents using the relation

$$\partial J^a + \partial^* J^{*a} = \partial_- J_+^a + \partial_+ J_-^a + 2i(\Box A^* T^a A) - 2i(A^* T^a \Box A),$$
$$\Box = \partial_+\partial_- - \partial\partial^*, \tag{2.21}$$

which holds on the tree level, and obtain

$$\mathcal{L}_1^{(3)} = \mathcal{L}_1^{(3-)} + \mathcal{L}_2^{(3-)},$$
$$\mathcal{L}_1^{(3-)} = \mathcal{L}_1^{(3+)} + \frac{g}{4}\left(\frac{\partial_-^2}{\partial\partial^*}\,J_+^a\right)\mathcal{A}_+^a,$$
$$\mathcal{L}_2^{(3-)} = i\,\frac{g}{2}\,\frac{\partial_-}{\partial\partial^*}\{[(\Box A^* T^a A) - (A^* T^a \Box A)] + \partial_-[\partial(J_s^a + J_2^a) - \partial^*(J_s^{*a} + J_2^{*a})]\}\mathcal{A}_+^a + \frac{g}{2}\,\frac{\partial_-}{\partial\partial^*}[\partial(J_s^a + J_2^a) + \partial^*(J_s^{*a} + J_2^{*a})]\mathcal{A}'^a. \tag{2.22}$$

Notice that the expression for $\mathcal{L}_1^{(3-)}$ does not change if we restrict the fields in the currents to the scattering modes A_s.

The quartic terms induced by t-channel exchange are

$$\langle \mathcal{L}_1^{(3+)}\mathcal{L}_1^{(3-)}\rangle_{A_t} + \langle \mathcal{L}_1^{(3+)}\mathcal{L}_2^{(3-)}\rangle_{A_t}. \tag{2.23}$$

The contraction $\langle\cdots\rangle_{A_t}$ simply means substituting in view of the kinetic term (2.8) the product of two exchanged fields according to

$$\langle \mathcal{A}_+^a \mathcal{A}_+^b\rangle_{A_t} \to -\frac{\delta_{ab}}{\partial_-^2}\left(1 + \frac{\partial_+\partial_-}{\partial\partial^*}\right), \quad \langle \mathcal{A}'^a \mathcal{A}'^b\rangle_{A_t} \to \delta_{ab}. \tag{2.24}$$

We add the first term in Eq. (2.23) to the original quartic terms $\mathcal{L}_{\rm scatt}^{(4)}$ (2.19) and obtain (disregarding total derivatives)

$$\langle \mathcal{L}_1^{(3+)}\mathcal{L}_1^{(3-)}\rangle_{A_t} + \mathcal{L}_{\rm scatt}^{(4)} = \frac{g^2}{8}\,J_-^a\,\frac{1}{\partial\partial^*}\,J_+^a - \frac{g^2}{16}\,\partial_+ J_-^a\,\frac{1}{\partial\partial^*}\,\partial_- J_+^a + g^2 j_s^a j_s^a - \frac{g^2}{2}\,j^a j^a - \frac{g^2}{4}\,j_D^r j_D^r, \tag{2.25}$$

where

$$j_s^a = j^a + \frac{i}{4}\,\frac{1}{\partial\partial^*}\,(\partial J^a - \partial^* J^{*a}) \tag{2.26}$$

is the current appearing already in $\mathcal{A}'$ channel in expression (2.12). We adopt the convention that the first current factor involves the fields describing the scattering gluons with large k_- and the second current the ones with large k_+. The form of this piece (2.25) coincides, up to the modifications to be discussed, with the effective action for quasielastic scattering. For later reference we write also the second term in expression (2.23) explicitly:

$$\langle \mathcal{L}_1^{(3+)}\mathcal{L}_2^{(3-)}\rangle_{A_t} = \frac{ig^2}{4}\,J_-^a\left(1 + \frac{1}{2}\,\frac{\partial_+\partial_-}{\partial\partial^*}\right) \times \frac{1}{\partial\partial^*}\left[\partial(J_s^a + J_2^a) - \partial^*(J_s^{*a} + J_2^{*a}) + \frac{1}{\partial_-}[(\Box A^* T^a A) - (A^* T^a \Box A)]\right] - \frac{g^2}{2}\,j_s^a\,\frac{\partial_-}{\partial\partial^*}[\partial(J_s^a + J_2^a) + \partial^*(J_s^{*a} + J_2^{*a})]. \tag{2.27}$$

III. INTEGRATION OVER HEAVY MODES

The triple vertices $\mathcal{L}_1^{(3)}$ induce further quartic terms by contracting two of them with an intermediate virtual gluon in the heavy mode A_1. Clearly also these terms contribute to high-energy quasielastic scattering. Assuming as above that two fields in the triple vertices (2.12) carry large k_- and restricting to the accuracy $\mathcal{O}(k_-^0)$ we can write these terms as

$$\langle \mathcal{L}_1^{(3+)}\mathcal{L}_1^{(3+)}\rangle_{A_1}. \tag{3.1}$$

The other terms in $\mathcal{L}_1^{(3)}$ (2.12) contribute only to order $\mathcal{O}(k_-^{-1})$.

The relevant part of the action for obtaining these quartic terms is the kinetic term of the A_1 modes and the triple terms $\mathcal{L}_1^{(3+)}$ linearized in A_1

$$-2A_1^{*a}\partial_+\partial_-\left(1 - \frac{\partial\partial^*}{\partial_+\partial_-}\right)A_1^a + \mathcal{L}_1^{(3+)}. \tag{3.2}$$

The result of the approximate integration over A_1 in the functional integral can be expressed by the value of this action at the saddle point given by the Lagrangian term

$$2A^{*a}_{1C}\partial_+\partial_-\left(1-\frac{\partial\partial^*}{\partial_+\partial_-}\right)A^a_{1C}. \tag{3.3}$$

Here A_{1C} is the solution of the linearized equation of motion derived from Eq. (3.2):

$$\begin{aligned}A_{1C}&=\frac{ig}{2}\left(\frac{1}{\partial_+}\tilde{\mathcal{A}}_{+t}T^aA_s\right)-\frac{ig}{2}\left(\frac{\partial_-}{\partial_+}\tilde{\mathcal{A}}_{+t}T^a\frac{1}{\partial_-}A_s\right)\\&\quad-\frac{ig}{2}\left[\frac{1}{\partial_+^2}\tilde{\mathcal{A}}_{+t}T^a\left(1-\frac{\partial\partial^*}{\partial_+\partial_-}\right)\partial_+A_s\right]\\&\quad+\frac{ig}{2}\left\{\left(\frac{\partial\partial^*}{\partial_+^2}\tilde{\mathcal{A}}_{+t}T^a\frac{1}{\partial_-}A_s\right)+\left(\frac{\partial^*}{\partial_+^2}\tilde{\mathcal{A}}_{+t}T^a\frac{\partial}{\partial_-}A_s\right)\right.\\&\quad\left.+\left(\frac{\partial}{\partial_+^2}\tilde{\mathcal{A}}_{+t}T^a\frac{\partial^*}{\partial_-}A_s\right)\right\}\\&\quad+\frac{g}{2}\left(\frac{1}{\partial_+}\tilde{\mathcal{A}}'_tT^a\frac{1}{\partial_-}A_s\right)-\frac{ig}{2}\left(\frac{1}{\partial_+}\tilde{A}^*_tT^a\frac{\partial^*}{\partial_-}A_s\right)\\&\quad-\frac{ig}{2}\left(\frac{1}{\partial_+}\tilde{A}_tT^a\frac{\partial}{\partial_-}A_s\right).\end{aligned}\tag{3.4}$$

The leading contribution at large k_- arises only from the first term. We neglect terms contributing to $\mathcal{O}(k_-^{-1})$ and use the free equations of motion $\Box A_s=0$ for the scattering modes.

The two fields in A_s modes in the result (3.3) can be expressed in terms of the currents introduced above. To illustrate how this works we pick up the terms where the leading term in Eq. (3.4) is multiplied with the one involving $\tilde{\mathcal{A}}'_t$:

$$\begin{aligned}&-\frac{ig^2}{2}\left\{\left(A_s^*T^a\frac{1}{\partial_+}\tilde{\mathcal{A}}_{+t}\right)\partial_+\partial_-\left(\frac{1}{\partial_+}\tilde{\mathcal{A}}'_tT^a\frac{1}{\partial_-}\mathcal{A}_s\right)\right.\\&\left.-\left(\frac{1}{\partial_-}\mathcal{A}_s^*T^a\frac{1}{\partial_+}\tilde{\mathcal{A}}'_t\right)\partial_+\partial_-\left(\frac{1}{\partial_+}\tilde{\mathcal{A}}_{+t}T^aA_s\right)\right\}.\end{aligned}\tag{3.5}$$

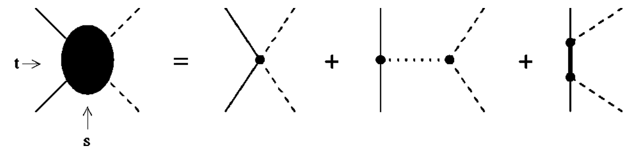

FIG. 3. The graphical illustration of Eq. (4.1) for the complete quartic terms. Different line forms represent different modes. Full line: scattered modes, dotted line: exchange modes, bold line: heavy modes, dashed line: the sum of all modes.

We take into account that in the considered part of the mode separation $\partial_-\tilde{\mathcal{A}}_t$ is small compared to ∂_-A_s but $\partial_+\tilde{\mathcal{A}}_t$ is large compared to ∂_+A_s and obtain

$$\begin{aligned}&-\frac{ig^2}{2}\left[\left(A_s^*T^a\frac{1}{\partial_+}\tilde{\mathcal{A}}_{+t}\right)(\tilde{\mathcal{A}}'_tT^aA_s)-\left(\frac{1}{\partial_-}A_s^*T^a\frac{1}{\partial_+}\tilde{\mathcal{A}}'_t\right)\right.\\&\left.\times(\tilde{\mathcal{A}}_{+t}T^a\partial_-A_s)\right].\end{aligned}\tag{3.6}$$

In this approximation we transform the expression (3.6) by partial integration

$$\begin{aligned}&-\frac{ig^2}{2}\left[\left(A_s^*T^a\frac{1}{\partial_+}\tilde{\mathcal{A}}_{+t}\right)(\tilde{\mathcal{A}}'_tT^aA_s)\right.\\&\left.-(A_s^*T^a\tilde{\mathcal{A}}'_t)\left(\frac{1}{\partial_+}\tilde{\mathcal{A}}_{+t}T^aA_s\right)\right].\end{aligned}\tag{3.7}$$

Using the relation (2.16) for the generators we obtain finally

$$\frac{ig^2}{2}(A_s^*T^aA_s)\left(\frac{1}{\partial_+}\tilde{\mathcal{A}}_{+t}T^a\tilde{\mathcal{A}}'_t\right)=\frac{ig^2}{2}j^a\left(\frac{1}{\partial_+}\tilde{\mathcal{A}}_{+t}T^a\tilde{\mathcal{A}}'\right).\tag{3.8}$$

Writing the current involving the large k_- scattering modes as the first factor we can suppress all signs referring to the modes.

In this way we obtain, for the quartic terms induced by the approximate heavy mode integration,

$$\begin{aligned}\langle\mathcal{L}_1^{(3+)}\mathcal{L}_1^{(3+)}\rangle_{A_1}&=\frac{ig^2}{8}J_-^a\left(\frac{1}{\partial_+}\mathcal{A}_+T^a\mathcal{A}_+\right)+\frac{g^2}{16}\frac{1}{\partial\partial^*}(\partial J^a+\partial^*J^{*a})\left\{i\left(\frac{1}{\partial_+}\mathcal{A}_+T^a\Box\frac{1}{\partial_+}\mathcal{A}_+\right)+i\left(\frac{\partial}{\partial_+}\mathcal{A}_+T^a\Box\frac{1}{\partial\partial_+}\mathcal{A}_+\right)\right.\\&\quad\left.+\left(\frac{\partial}{\partial_+}\mathcal{A}_+T^a\frac{1}{\partial}\mathcal{A}'\right)+\text{c.c.}\right\}+\frac{ig^2}{2}j_s^a\left(\frac{1}{\partial_+}\mathcal{A}_+T^a\mathcal{A}'\right)+i\,\frac{g^2}{16}\frac{1}{\partial\partial^*}(\partial J^a-\partial^*J^{*a})\left\{i\left(\frac{\partial}{\partial_+}\mathcal{A}_+T^a\frac{1}{\partial}\mathcal{A}'\right)\right.\\&\quad\left.-\left(\frac{\partial}{\partial_+}\mathcal{A}_+T^a\Box\frac{1}{\partial\partial_+}\mathcal{A}_+\right)+\text{c.c.}\right\}+\frac{g^2}{8}j_D^r\left\{\left(\frac{\partial}{\partial_+}\mathcal{A}_+D^r\Box\frac{1}{\partial\partial_+}\mathcal{A}_+\right)-i\left(\frac{\partial}{\partial_+}\mathcal{A}_+D^r\frac{1}{\partial}\mathcal{A}'\right)+\text{c.c.}\right\}.\end{aligned}\tag{3.9}$$

The fact that the two fields in A_s modes carrying large k_- can be factorized in terms of the currents is the first sign of factorizability in quasielastic scattering which is essential for identifying the perturbative Reggeons.

IV. QUASIELASTIC SCATTERING

A. Quartic terms

Summarizing the results of the previous two sections we write down the sum of quartic terms, where two of the fields are in scattering modes with large k_- and the two others in all modes but with small k_- (see Fig. 3):

$$\mathcal{L}_{\rm tot}^{(4)} = \langle \mathcal{L}_1^{(3+)}(\mathcal{L}_1^{(3-)} + \mathcal{L}_2^{(3-)})\rangle_{A_t} + \langle \mathcal{L}_1^{(3+)}\mathcal{L}_1^{(3+)}\rangle_{A_1} + \mathcal{L}_{\rm scatt}^{(4)}. \tag{4.1}$$

To avoid misunderstanding we say that $\mathcal{L}_{\rm tot}^{(4)}$ does not mean all quartic terms but a set of terms complete in the sense that it allows to extract the vertices of the effective action.

We have observed that the two fields in scattering modes factorize in terms of the currents. We shall first study terms where this current is J_-^a or $(\partial J^a + \partial^* J^{*a})$. The latter can be replaced by $\partial_+ J_-^a$ [compare Eq. (2.21)]. These terms correspond to the exchange of $\mathcal{A}_{t+}$.

The terms with currents j^a or j_s^a [Eq. (2.26)] are related to the exchange of $\mathcal{A}_t'$. Further there are terms with j_D^r which correspond to the representation symmetric in the color indices in the t channel, the generators of which we have denoted by D^r (2.18).

In each case we shall consider in particular the contribution to elastic scattering obtained by restricting also the other two fields to the scattering modes. In this way we shall arrive at the effective action $\mathcal{L}_{\rm eff,scatt}$ describing high-energy quasielastic scattering.

B. $\mathcal{A}_+$ channel

From Eq. (4.1) supplemented by Eqs. (2.25), (2.27), and (3.9) we obtain, disregarding total derivatives,

$$\begin{aligned}
\mathcal{L}_{\rm tot}^{(4)}|_{\mathcal{A}_+} \\
= \frac{g^2}{8} J_-^a \Bigg\{ \frac{1}{\partial\partial^*}\left(1+\frac{1}{2}\frac{\partial_+\partial_-}{\partial\partial^*}\right) \{ J_+^a + 2i[\partial(J_s^a + J_2^a) \\
- \partial^*(J_s^{*a} + J_2^{*a})] + 2i\frac{1}{\partial_-}[(\Box A^* T^a A) - (A^* T^a \Box A)]\} \\
+ \frac{1}{2}\left(\frac{\partial_+}{\partial\partial^*}\right)^2 J_-^a + i\left(\frac{1}{\partial_+}\mathcal{A}_+ T^a \mathcal{A}_+\right) \\
- \frac{1}{2}\frac{\partial_+}{\partial\partial^*}\left[i\left(\frac{\partial}{\partial_+}\mathcal{A}_+ T^a \Box \frac{1}{\partial\partial_+}\mathcal{A}_+\right)\right. \\
\left. + i\left(\frac{1}{\partial_+}\mathcal{A}_+ T^a \Box \frac{1}{\partial_+}\mathcal{A}_+\right) + \left(\frac{\partial}{\partial_+}\mathcal{A}_+ T^a \frac{1}{\partial}\mathcal{A}'\right) + \text{c.c.}\right]\Bigg\}.
\end{aligned} \tag{4.2}$$

If we restrict all fields to the scattering modes, where we have $\Box A_s = 0$, a number of terms vanishes obviously. We shall see that the contribution to quasielastic scattering is given by the first term only with the substitution of J_+^a by

$$J_{R+}^a = i(A_R^* T^a \overleftrightarrow{\partial}_+ A_R), \quad A_R^a = -\frac{\partial^*}{\partial} A^{*a}. \tag{4.3}$$

We can say that all the other terms just result in dressing the "bare" current to become J_{R+}^a. Notice that for A_s modes the relation between A_R^a and A^a is just the gauge transformation which leads from the gauge $A_-^a = 0$, which is used here, to the gauge $A_+^a = 0$. Clearly this structure of the result is just what should be expected from parity symmetry, which implies the symmetry under the exchange of indices + and − and the gauge transformation (4.3):

$$+ \leftrightarrow -, \quad A \to A_R. \tag{4.4}$$

The contribution to quasielastic scattering with $\mathcal{A}_+$ exchange can be written as

$$\mathcal{L}_{\rm eff,scatt}^{(4)}|_{\mathcal{A}_+} = \frac{g^2}{8} J_-^a \frac{1}{\partial\partial^*} J_{R+}^a - \frac{g^2}{16}(\partial_+ J_-^a)\frac{1}{(\partial\partial^*)^2}\partial_- J_{R+}^a. \tag{4.5}$$

To prove this assertion it is convenient to use the following "dressing relations," leading to the replacement of the current J_+ by the "dressed" one J_{R+}:

$$\begin{aligned}
J_+^a + i\partial\partial^*\left(\frac{1}{\partial_+}\mathcal{A}_+ T^a \mathcal{A}_+\right) + 2i[\partial(J_s^a + J_2^a) - \partial^*(J_s^{*a} + J_2^{*a})] \\
= J_{R+}^a - i\partial_+\left[\left(\partial_-\mathcal{A}_+ T^a \frac{1}{\partial_+}\mathcal{A}_+\right) - \left(\frac{\partial}{\partial^*} A T^a A\right)\right. \\
\left. - \left(\frac{\partial^*}{\partial} A^* T^a A^*\right)\right],
\end{aligned} \tag{4.6}$$

$$\frac{i}{2}\left(\frac{\partial^*}{\partial_+}\mathcal{A}_+ T^a \frac{1}{\partial^*}\mathcal{A}'\right) - \frac{i}{2}\left(\frac{\partial}{\partial_+}\mathcal{A}_+ T^a \frac{1}{\partial}\mathcal{A}'\right) = \left(\frac{\partial}{\partial^*} A T^a A\right) + \left(\frac{\partial^*}{\partial} A^* T^a A^*\right), \tag{4.7}$$

$$\begin{aligned}
i(\partial_+ J_-^a + \partial_- J_+^a) - 2\partial_-[\partial(J_s^a + J_2^a) - \partial^*(J_s^{*a} + J_2^{*a})] \\
+ 2\partial\partial^*\left(\partial_-\mathcal{A}_+ T^a \frac{1}{\partial_+}\mathcal{A}_+\right) = i(\partial_+ J_{R-}^a + \partial_- J_{R+}^a) \\
+ 2\partial_+\left[\partial\left(\partial_- A T^a \frac{1}{\partial_+}\mathcal{A}_+\right) + \partial^*\left(\partial_- A^* T^a \frac{1}{\partial_+}\mathcal{A}_+\right)\right].
\end{aligned} \tag{4.8}$$

The relations hold only if all fields involved are in the A_s modes (which we have not indicated explicitely to simplify the notations). They are derived by straightforward calculations, where the condition $\Box A_s=0$ is used repeatedly. With these relations applied to Eq. (4.2) we obtain Eq. (4.5) immediately.

C. $\mathcal{A}'$ channel

From Eq. (4.1) supplemented by Eqs. (2.25), (2.27), and (3.9) we obtain

$$\begin{aligned}\mathcal{L}_{\rm tot}^{(4)}|_{\mathcal{A}'}&\\ &=g^2 j_s^a\bigg\{ j_s^a-\frac{1}{2}\frac{\partial_-}{\partial\partial^*}[\partial(J_s^a+J_2^a)+\partial^*(J_s^{*a}+J_2^{*a})]\\ &\quad+\frac{i}{2}\left(\frac{1}{\partial_+}\mathcal{A}_+T^a\mathcal{A}'\right)\bigg\}-\frac{g^2}{2}j^aj^a\\ &\quad+\frac{g^2}{4}(j_s^a-j^a)\bigg\{ i\left(\frac{\partial}{\partial_+}\mathcal{A}_+T^a\frac{1}{\partial}\mathcal{A}'\right)\\ &\quad-\left(\frac{\partial}{\partial_+}\mathcal{A}_+T^a\Box\frac{1}{\partial\partial_+}\mathcal{A}_+\right)+\text{c.c.}\bigg\}.\end{aligned}\tag{4.9}$$

If we restrict all fields to the scattering modes we obtain a simple expression where the two fields with small k_- can also be expressed in currents j_{Rs}^a and j_R^a. These currents are given by the expressions for j_s^a and j^a with substituting A^a by A_R^a in analogy to Eq. (4.3).

As in the previous subsection it is convenient to use ''dressing relations'' which hold if all fields involved are in the A_s modes, with $\Box A_s=0$:

$$\begin{aligned}2j_s^a-j^a+i\left(\frac{1}{\partial_+}\mathcal{A}_+T^a\mathcal{A}'\right)-\frac{1}{2}\frac{\partial_-}{\partial\partial^*}[\partial(J_s^a+J_2^a)&\\ +\partial^*(J_s^{*a}+J_2^{*a})]=2j_{Rs}^a-j_R^a,&\end{aligned}\tag{4.10}$$

$$j^a+\frac{i}{2}\left[\left(\frac{\partial}{\partial_+}\mathcal{A}_+T^a\frac{1}{\partial}\mathcal{A}'\right)+\text{c.c.}\right]=j_R^a.\tag{4.11}$$

Applying these relations to Eq. (4.9) we obtain that quasielastic scattering with $\mathcal{A}'$ exchange is described by

$$\mathcal{L}_{\rm eff,scatt}|_{\mathcal{A}'}=g^2j_s^aj_{Rs}^a-\frac{g^2}{2}j^aj_R^a.\tag{4.12}$$

D. Symmetric gauge group channel

From Eq. (4.1) supplemented by Eqs. (2.25) and (3.9) we have

$$\begin{aligned}\mathcal{L}_{\rm tot}^{(4)}|_D=-\frac{g^2}{4}j_D^r\bigg\{ j_D^r-\frac{1}{2}\bigg[\left(\frac{\partial}{\partial_+}\mathcal{A}_+D^r\Box\frac{1}{\partial\partial_+}\mathcal{A}_+\right)&\\ -i\left(\frac{\partial}{\partial_+}\mathcal{A}_+D^r\frac{1}{\partial}\mathcal{A}'\right)+\text{c.c.}\bigg]\bigg\}.&\end{aligned}\tag{4.13}$$

As in the other cases we study the special configuration where all fields are in the modes A_s. Now we have, as the ''dressing relation,''

$$j_D^r+\frac{1}{2}\left[i\left(\frac{\partial}{\partial_+}\mathcal{A}_+D^r\frac{1}{\partial}\mathcal{A}'\right)+\text{c.c.}\right]=j_{RD}^r\tag{4.14}$$

and obtain as the symmetric gauge-group channel contribution to quasielastic scattering

$$\mathcal{L}_{\rm eff,scatt}|_D=-\frac{g^2}{4}j_D^rj_{RD}^r.\tag{4.15}$$

E. Effective action for quasielastic scattering

The high-energy scattering of gluons at the tree level can be effectively described by the sum of terms (4.5), (4.12), (4.15) up to corrections of order $\mathcal{O}(s^{-1})$. It is remarkable that we have a sum of terms factorized with respect to the t channel. Parity symmetry implies that the currents describing the scattering at large k_- have their counterparts in the case of scattering at large k_+ obtained from the former by replacing indices $-\to+$ and by the gauge transformation $A\to A_R$ (4.4). Due to this symmetry the result has always the bilinear form of products of currents factorizable in the t channel. It is enough to study the triple terms $\mathcal{L}_1^{(3)}$ describing large k_- scattering and the quartic terms of the original action $\mathcal{L}_{\rm scatt}^{(4)}$ to find out what kind of terms arise in the effective action of high-energy quasielastic scattering.

Indeed we see that the quartic terms $\langle\mathcal{L}_1^{(3+)}\mathcal{L}_1^{(3-)}\rangle_{A_t}+\mathcal{L}_{\rm scatt}^{(4)}$ (2.25) coincide with the ones in Eqs. (4.5), (4.12), and (4.15) up to the index R at the second current indicating the substitution $A\to A_R$. This dressing is the only effect of the contribution from the heavy mode intermediate state and of terms in $\mathcal{L}_1^{(3)}$ negligible in the large k_- scattering. The lengthy explicit expression for the complete quartic terms $\mathcal{L}_{\rm tot}^{(4)}$ is necessary only in the derivation of the effective vertices of gluon production discussed in the next section.

We introduce five pairs of pre-Reggeon fields, one pair for each current times current term. In this way the quasielastic high-energy scattering can be described by the following effective action:

$$\begin{aligned}\mathcal{L}_{\rm eff,scatt}&=\mathcal{L}_{\rm kin}+\mathcal{L}_{s-}+\mathcal{L}_{s+},\\ \mathcal{L}_{\rm kin}&=-2A_s^{a*}(\partial_-\partial_+-\partial\partial^*)A_s^a-2\mathcal{A}_+^a\partial\partial^*\mathcal{A}_-^a\\ &\quad+\mathcal{A}_{(+)}^a\mathcal{A}_{(-)}^a\\ &\quad-\mathcal{A}_s'^{a(+)}\mathcal{A}_s'^{a(-)}+2\mathcal{A}_2'^{a(+)}\mathcal{A}_2'^{a(-)}+B^{r(+)}B^{r(-)},\end{aligned}\tag{4.16}$$

$$\begin{aligned}\mathcal{L}_{s-}=&-\frac{g}{2}J_-^a\mathcal{A}_+^a-\frac{g}{4}\left(\frac{\partial_+}{\partial\partial^*}J_-^a\right)\mathcal{A}_{(+)}^a-gj_s^a\mathcal{A}_s'^{a(+)}\\ &-gj^a\mathcal{A}_2'^{a(+)}-\frac{g}{2}j_D^rB^{r(+)}.\end{aligned}$$

$\mathcal{L}_{s+}$ is obtained from $\mathcal{L}_{s-}$ by replacing the labels $+\leftrightarrow-$ and the currents by their partners with label R. This result is checked simply by observing that each of the quartic scattering terms (4.5), (4.12), (4.15) is reproduced from Eq. (4.16) by integrating out the corresponding pre-Reggeon.

Each pre-Reggeon is represented by a pair of fields. The distinction by the labels $+$ and $-$ is related to the fact that the Reggeon exchange is oriented in rapidity: The pre-Reggeon fields with label $+$ couple to scattering gluons with large k_- only and vice versa.

The pre-Reggeon $(\mathcal{A}_+^a, \mathcal{A}_-^a)$ is the leading one. It contributes as s^1 to the amplitude and it is well known that the exchange of two of them, interacting via emission and absorption of s-channel gluons, results in the BFKL Pomeron. The pre-Reggeon $(\mathcal{A}_{(+)}^a, \mathcal{A}_{(-)}^a)$ can be regarded as the nonleading partner of the first one. Both conserve helicity of the scattering gluon and both carry positive P parity and negative C parity.

In the $\mathcal{A}'$ channel, considered in the Sec. IV C we encounter two pre-Reggeons at the nonleading level $\mathcal{O}(s^0)$, $\mathcal{A}_s'^{a(\pm)}$, and $\mathcal{A}_2'^{a(\pm)}$. Both conserve helicity but their vertices depend on the sign of the helicity. They carry negative P and C parities. The difference between them is not obvious. At this point we just notice that the coupling of $\mathcal{A}_s'^{a(\pm)}$ involves a term sensitive to the transverse momenta of the scattering gluons whereas $\mathcal{A}_2'^{a}$ is insensitive to transverse momenta.

The pre-Reggeons discussed so far are in the gauge-group state of the gluon. The t-channel factorization unavoidably leads also to exchanges in other gauge-group representations, which arise as the symmetric part in the tensor product of two adjoint representations. Among the corresponding pre-Reggeons there is a color singlet one.

V. INELASTIC SCATTERING

A. Terms of order 5

The essential ingredients of the high-energy effective action beyond the effective action of quasielastic scattering are the production vertices. In general these vertices can be obtained from the effective interaction terms of order 5, where the fields are in scattering modes A_s and their longitudinal momenta correspond to the kinematics of an inelastic $2\to 3$ scattering in multi-Regge kinematics (2.6), which we denote by $\mathcal{L}^{(5)}$. The production vertices, triple vertices involving two pre-Reggeons and one scattering particle, are obtained by factorizing the scattering vertices $\mathcal{L}_{s-}$ and $\mathcal{L}_{s+}$:

$$\mathcal{L}^{(5)} = \langle \mathcal{L}_{s-} \mathcal{L}_p \mathcal{L}_{s+} \rangle. \tag{5.1}$$

Here the brackets stand for contraction of the pre-Reggeons by substituting their product by propagators read off from the kinetic terms in $\mathcal{L}_{\text{eff,scatt}}$ (4.16).

A contribution to $\mathcal{L}^{(5)}$ in the desired mode configuration is obtained from $\mathcal{L}_{\text{tot}}^{(4)}$ by contracting with $\mathcal{L}_1^{(3-)}$, $\langle \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_1^{(3-)} \rangle_{A_t}$. $\mathcal{L}_{\text{tot}}^{(4)}$ involves two fields in A_s mode with large k_- and $\mathcal{L}_1^{(3-)}$ two fields in A_s mode with large k_+. One of the two other fields in $\mathcal{L}_{\text{tot}}^{(4)}$ is to be restricted to the scattering mode and the other to the exchange mode.

In this contribution the factorization in the two t channels is immediate, because the pair of fields with large k_- is factorized in $\mathcal{L}_{\text{tot}}^{(4)}$ (4.2), (4.9), (4.13) and the pair of fields with large k_+ is factorized in $\mathcal{L}_1^{(3-)}$. Therefore the production vertices involving only those pre-Reggeons the couplings (currents) of which appear in $\mathcal{L}_1^{(3\pm)}$ can be read off directly from $\mathcal{L}_{\text{tot}}^{(4)}$. This does not apply only to one of the odd-parity Reggeons $\mathcal{A}'^{\pm}$ and to the gauge-group symmetric pre-Reggeons $B^{r(\pm)}$.

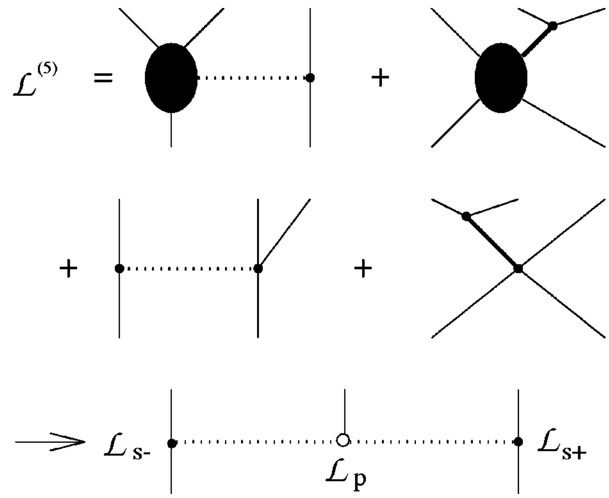

FIG. 4. The graphical illustration of the terms describing inelastic scattering Eq. (5.2) and of the factorization leading to the production vertices Eq. (5.1).

Besides of the terms discussed we have more contributions. To order 5 the interaction terms in the mode configuration of interest are given by (Fig. 4)

$$\mathcal{L}^{(5)} = \langle \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_1^{(3-)} \rangle_{A_t} + \langle \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_2^{(3-)} \rangle_{A_t} + \langle \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_1^{(3+)} \rangle_{A_1} + \langle \mathcal{L}_1^{(3+)} \mathcal{L}_{\text{scatt}}^{(4)} \rangle_{A_t} + \langle \mathcal{L}_1^{(3+)} \mathcal{L}_{\text{scatt}}^{(4)} \rangle_{A_1}. \tag{5.2}$$

In Sec. IV we have seen that the heavy mode contributions and the contribution with $\mathcal{L}_2^{(3-)}$ merely lead to the "dressing" of the currents involving the large k_+ scattering modes. Analogously, the second and third terms in Eq. (5.2) contribute only to dressing the currents in $\mathcal{L}_{s+}$ (4.16). The structure of the result appears already in the sum of the other terms and the effective production vertices can be obtained without writing the second and the third terms in Eq. (5.2) explicitly.

B. $\mathcal{A}_- - \mathcal{A}_+$ exchanges

As discussed above the production vertices involving the even-parity pre-Reggeons $\mathcal{A}_\pm$ and $\mathcal{A}_{(\pm)}$ can be read off directly from the complete quartic terms $\mathcal{L}_{\text{tot}}^{(4)}|_{\mathcal{A}_+}$ (4.2). We recall that J_-^a involves the two scattering fields with large k_-. One of the other two fields has to be restricted to the scattering modes too. We may restrict ourselves to the contributions where the inelastic gluon corresponds to the field A_s^*. The production vertex with A_s is then obtained by complex conjugation. The remaining field in $\mathcal{L}_{\text{tot}}^{(4)}$ is to be projected onto $\mathcal{A}_+$ in the t-channel modes, because the wanted contribution $\langle \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_1^{(3-)} \rangle$ to $\mathcal{L}_p^{(5)}$ is obtained by contraction with $\mathcal{L}_1^{(3-)}|_{\mathcal{A}_+} = g/4[(\partial_-^2/\partial\partial^*) J_+^a] \mathcal{A}_{t+}^a$.

For illustration we pick up in $\mathcal{L}_{\rm tot}^{(4)}$ (4.2) the term $i(g^2/8)J_-^a[(1/\partial_+)\mathcal{A}_+T^a\mathcal{A}_+]$. Its contribution to the inelastic production with A_s^* is given by

$$i\,\frac{g^3}{32}\left\{\left[J_-T^a\left(1+\frac{\partial_+\partial_-}{\partial\partial^*}\right)\frac{1}{\partial\partial^*}\,J_+\right]\right.$$
$$\left.+\partial_+\left[J_-T^a\left(1+\frac{\partial_+\partial_-}{\partial\partial^*}\right)\frac{1}{\partial_+}\,\frac{1}{\partial\partial^*}\,J_+\right]\right\}\frac{1}{\partial}\,A_s^{*a}. \quad (5.3)$$

For the leading exchange related to $J_\pm$ the correction to the propagator $(1+\partial_+\partial_-/\partial\partial^*)$ has to be included. Up to terms contributing to order $\mathcal{O}(1/s)$ we obtain

$$i\,\frac{g^3}{16}\left\{2\left(J_-T^a\,\frac{1}{\partial\partial^*}\,J_+\right)+\left(\partial_+J_-T^a\,\frac{\partial_-}{(\partial\partial^*)^2}\,J_+\right)\right.$$
$$+2\partial_+\left(J_-T^a\,\frac{\partial_-}{(\partial\partial^*)^2}\,J_+\right)$$
$$\left.+\left(\partial_+J_-T^a\,\frac{1}{\partial\partial^*\partial_+}\,J_+\right)\right\}\frac{1}{\partial}\,A_s^{*a}. \quad (5.4)$$

The last term can be transformed in the following way:

$$\left(\partial_+J_-T^a\,\frac{1}{\partial\partial^*\partial_+}\,J_+\right)\frac{1}{\partial}\,\mathcal{A}_s^{*a}$$
$$=-\left(\partial_+J_-T^a\,\frac{1}{\partial\partial^*}\,J_+\right)\frac{\partial_-}{\partial^2\partial^*}\,A_s^{*a}$$
$$+\left(\partial_+^2J_-T^a\,\frac{1}{\partial\partial^*}\,J_+\right)\frac{1}{\partial_+^2\partial}\,A_s^{*a}. \quad (5.5)$$

The second term gives a small contribution. We have used the condition $\Box A_s=0$.

Factorizing $\mathcal{L}_{s\pm}$ we obtain the corresponding contribution to $\mathcal{L}_p$:

$$ig\left\{(\partial\partial^*\mathcal{A}_-T^a\mathcal{A}_+)-\frac{1}{2}\left(\partial\partial^*\mathcal{A}_{(-)}T^a\,\frac{1}{\partial\partial^*}\,\mathcal{A}_{(+)}\right)\right.$$
$$-\partial_+\left(\partial\partial^*\mathcal{A}_-T^a\,\frac{1}{\partial\partial^*}\,\mathcal{A}_{(+)}\right)$$
$$\left.-\frac{1}{2}\,\frac{\partial_-}{\partial\partial^*}\,(\partial\partial^*\mathcal{A}_{(-)}T^a\mathcal{A}_+)\right\}\frac{1}{\partial}\,A_s^{*a}. \quad (5.6)$$

Repeating the outlined procedure with all terms in $\mathcal{L}_{\rm tot}^{(4)}$ (4.2) we obtain the effective production vertices in the channels with the pre-Reggeons $\mathcal{A}_\pm$ and their nonleading partners $\mathcal{A}_{(\pm)}$:

$$\mathcal{L}_p|_{\mathcal{A}_\pm}=-\frac{ig}{2}\left\{\left[2(\partial^*\mathcal{A}_-T^a\partial\mathcal{A}_+)+\frac{1}{2}\,(\mathcal{A}_{(-)}T^a\partial\mathcal{A}_{(+)})\right.\right.$$
$$\left.+\frac{1}{2}\left(\frac{\partial^*}{\partial}\,\mathcal{A}_{(-)}T^a\,\frac{\partial}{\partial^*}\,\mathcal{A}_{(+)}\right)\right]\frac{1}{\partial}\,A^{*a}$$
$$+\left[\left(\frac{1}{\partial}\,\mathcal{A}_{(-)}T^a\partial\mathcal{A}_+\right)+\frac{1}{\partial\partial^*}\,(\partial^*\mathcal{A}_{(-)}T^a\partial\mathcal{A}_+)\right. \quad (5.7)$$
$$\left.+\frac{1}{\partial\partial^*}\,(\mathcal{A}_{(-)}T^a\partial\partial^*\mathcal{A}_+)\right]\frac{\partial_-}{\partial}\,A^{*a}$$
$$+\left[\left(\partial^*\mathcal{A}_-T^a\,\frac{1}{\partial^*}\,\mathcal{A}_{(+)}\right)+\frac{1}{\partial\partial^*}\,(\partial^*\mathcal{A}_-T^a\partial\mathcal{A}_{(+)})\right.$$
$$\left.\left.+\frac{1}{\partial\partial^*}\,(\partial\partial^*\mathcal{A}_-T^a\mathcal{A}_{(+)})\right]\frac{\partial_+}{\partial}\,A^{*a}\right\}+\text{c.c.}$$

C. $\mathcal{A}'-\mathcal{A}'$ exchange

We consider first the contribution in the term $\langle\mathcal{L}_{\rm tot}^{(4)}\mathcal{L}_1^{(3-)}\rangle|_{A_t}$ to these channels obtained by combining $\mathcal{L}_{\rm tot}^{(4)}|_{\mathcal{A}'}$ (4.9) with $\mathcal{L}_1^{(3-)}|_{\mathcal{A}'}=-gj_s^a\mathcal{A}'^a$ (2.22),

$$\langle\mathcal{L}_{\rm tot}^{(4)}\mathcal{L}_1^{(3-)}\rangle|_{\mathcal{A}'-\mathcal{A}'}$$
$$=-i\,\frac{g^3}{4}\left\{2\left(\frac{1}{\partial}\,j_sT^a\partial j_s\right)+\left(\partial^*j_sT^a\,\frac{1}{\partial^*}\,j_s\right)\right.$$
$$\left.+\left(\frac{\partial^*}{\partial}\,j_sT^a\,\frac{\partial}{\partial^*}\,j_s\right)+\partial^*\left((j_s-j)T^a\,\frac{1}{\partial^*}\,j_s\right)\right\}\frac{1}{\partial}\,A_s^{*a}+\text{c.c.}$$
$$(5.8)$$

Now we calculate the contribution of $\langle\mathcal{L}_1^{(3+)}\mathcal{L}_{\rm scatt}^{(4)}\rangle|_{A_t}$ using

$$\mathcal{L}_1^{(3+)}|_{\mathcal{A}'}=-gj_s^a\mathcal{A}'^a,\quad \mathcal{L}_{\rm scatt}^{(4)}|_{\mathcal{A}'}=-\frac{g^2}{2}\,j^aj^a. \quad (5.9)$$

One of the two fields in the first current factor in $\mathcal{L}_{\rm scatt}^{(4)}|_{\mathcal{A}'}$ corresponds to the produced particle and the other involves the t-channel modes to be contracted with $\mathcal{L}_1^{(3+)}|_{\mathcal{A}'}$,

$$\mathcal{L}_{\rm scatt}^{(4)}|_{\mathcal{A}'}\to\frac{ig^2}{4}\left(\frac{1}{\partial}\,\mathcal{A}'T^aj\right)A^{*a}+\text{c.c.} \quad (5.10)$$

This implies

$$\langle\mathcal{L}_1^{(3+)}\mathcal{L}_{\rm scatt}^{(4)}\rangle_{A_t}|_{\mathcal{A}'-\mathcal{A}'}=-\frac{ig^3}{4}\left(\frac{1}{\partial}\,j_sT^aj\right)A^{*a}+\text{c.c.}$$
$$(5.11)$$

The next term to be evaluated is $\langle\mathcal{L}_{\rm scatt}^{(4)}\mathcal{L}_1^{(3+)}\rangle_{A_1}$ supplying the bremsstrahlung correction to the original quartic vertex caused by the leading term in $\mathcal{L}_1^{(3)}$ (2.12). In Sec. III we have done the calculation for the heavy mode contributions $\mathcal{L}_1^{(3)}|_{\mathcal{A}'}=-gj^a\mathcal{A}'^a$ resulting in Eq. (3.8). The heavy mode contribution involving the quartic vertex $\mathcal{L}_{\rm scatt}^{(4)}|_{\mathcal{A}'}$ (5.9) is given by the analogous expression with the substitution $\mathcal{A}'^a\to(g/2)j^a$. This gives immediately the result

$$\langle\mathcal{L}_{\rm scatt}^{(4)}\mathcal{L}_1^{(3+)}\rangle_{A_1}=-\frac{ig^3}{4}\,(jT^aj)\,\frac{1}{\partial}\,A^{*a}+\text{c.c.} \quad (5.12)$$

For the sum of the calculated contributions we have

$$(\langle \mathcal{L}_{\rm tot}^{(4)} \mathcal{L}_1^{(3-)} \rangle_{A_t} + \langle \mathcal{L}_1^{(3+)} \mathcal{L}_{\rm scatt}^{(4)} \rangle_{A_t} + \langle \mathcal{L}_{\rm scatt}^{(4)} \mathcal{L}_1^{(3+)} \rangle_{A_1})|_{\mathcal{A}'-\mathcal{A}'}$$
$$= -\frac{ig^3}{4} \left\{ 2\left(\frac{1}{\partial} j_s T^a \partial j_s \right) + 2\left(\partial^* j_s T^a \frac{1}{\partial^*} j_s \right) + \left(\frac{\partial^*}{\partial} j_s T^a \frac{\partial}{\partial^*} j_s \right) + (j_s T^a j_s) + (j T^a j) - \left(\frac{1}{\partial} j_s T^a \partial j_s \right) - (j_s T^a j) - \left(\partial^* j T^a \frac{1}{\partial^*} j_s \right) - (j T^a j_s) \right\} \frac{1}{\partial} A^{*a} + \text{c.c.} \tag{5.13}$$

The factorization with respect to $\mathcal{L}_{s\pm}$ amounts to the replacement $-g j_s^a \to \mathcal{A}_s'^{\,a}$, $g/2 j^a \to \mathcal{A}'^{\,a}$ and we obtain the effective production vertices

$$\mathcal{L}_p|_{\mathcal{A}'-\mathcal{A}'}$$
$$= -\frac{ig}{4} \left\{ 2\left(\frac{1}{\partial} \mathcal{A}_s'^{(-)} T^a \partial \mathcal{A}_s'^{(+)} \right) + 2\left(\partial^* \mathcal{A}_s'^{(-)} T^a \frac{1}{\partial^*} \mathcal{A}_s'^{(+)} \right) + \left(\frac{\partial^*}{\partial} \mathcal{A}_s'^{(-)} T^a \frac{\partial}{\partial^*} \mathcal{A}_s'^{(+)} \right) + (\mathcal{A}_s'^{(-)} T^a \mathcal{A}_s'^{(+)}) + 4(\mathcal{A}_2'^{(-)} T^a \mathcal{A}_2'^{(+)}) + 2\left(\frac{1}{\partial} \mathcal{A}_s'^{(-)} T^a \partial \mathcal{A}_2'^{(+)} \right) + 2(\mathcal{A}_s'^{(-)} T^a \mathcal{A}_2'^{(+)}) + 2\left(\partial^* \mathcal{A}_2'^{(-)} T^a \frac{1}{\partial^*} \mathcal{A}_s'^{(+)} \right) + 2(\mathcal{A}_2'^{(-)} T^a \mathcal{A}_s'^{(+)}) \right\} \frac{1}{\partial} A^{*a} + \text{c.c.} \tag{5.14}$$

D. $\mathcal{A}'-\mathcal{A}_+$ exchanges

Unlike the previous case the production vertices can be obtained from the quartic terms $\mathcal{L}_{\rm tot}^{(4)}|_{\mathcal{A}'}$ (4.9). We calculate the part $\langle \mathcal{L}_{\rm tot}^{(4)} \mathcal{L}_1^{(3-)} \rangle$ obtained by contracting the quartic terms with $\mathcal{L}_1^{(3-)}|_{\mathcal{A}_+}$:

$$\langle \mathcal{L}_{\rm tot}^{(4)} \mathcal{L}_1^{(3-)} \rangle|_{\mathcal{A}'-\mathcal{A}_+}$$
$$= \frac{g^3}{8} \left\{ \partial_- \left(\frac{1}{\partial} j_s T^a \frac{1}{\partial^*} J_+ \right) + \frac{\partial_-}{\partial} \left(j_s T^a \frac{1}{\partial^*} J_+ \right) - \frac{\partial_-}{\partial} \left(j T^a \frac{1}{\partial^*} J_+ \right) + \frac{1}{2} \partial^* \left((j_s - j) T^a \frac{\partial_-}{\partial^{*2} \partial} J_+ \right) + \frac{1}{2} \partial \left(\frac{\partial^*}{\partial} j_s T^a \frac{\partial_-}{\partial^{*2} \partial} J_+ \right) \right\} \frac{1}{\partial} A_s^{*a} + \text{c.c.} \tag{5.15}$$

The remaining contributions in Eq. (5.2) lead merely to the "dressing" substitution $J_+ \to J_{R_+}$. The corresponding contribution to the effective production vertices is obtained factorizing $\mathcal{L}_{s+}|_{\mathcal{A}'}$ and $\mathcal{L}_{s-}|_{\mathcal{A}_+}$. The result is obtained from Eq. (5.15) by the following substitutions:

$$-g j_s^a \to \mathcal{A}_s'^{\,a}, \quad \frac{g}{2} j^a \to \mathcal{A}'^{\,a},$$

$$-\frac{g}{4} \frac{1}{\partial \partial^*} J_{R+}^a \to \mathcal{A}_+^a, \quad \frac{g}{4} \frac{\partial_-}{\partial \partial^*} J_{R+}^a \to \mathcal{A}_{(+)}^a,$$

$$\mathcal{L}_p|_{\mathcal{A}'-\mathcal{A}_+} = -\frac{g}{2} \left\{ -\partial_- \left(\frac{1}{\partial} \mathcal{A}_s'^{(-)} T^a \partial \mathcal{A}_+ \right) - \frac{\partial_-}{\partial} (\mathcal{A}_s'^{(-)} T^a \partial \mathcal{A}_+) - 2 \frac{\partial_-}{\partial} (\mathcal{A}_2'^{(-)} T^a \partial \mathcal{A}_+) + \partial^* \left(\mathcal{A}_2'^{(-)} T^a \frac{1}{\partial^*} \mathcal{A}_{(+)} \right) + \frac{1}{2} \partial^* \left(\mathcal{A}_s'^{(-)} T^a \frac{1}{\partial^*} \mathcal{A}_{(+)} \right) + \frac{1}{2} \partial \left(\frac{\partial^*}{\partial} \mathcal{A}_s'^{(-)} T^a \frac{1}{\partial^*} \mathcal{A}_{(+)} \right) \right\} \frac{1}{\partial} A_s^{*a} + \text{c.c.} \tag{5.16}$$

The case where the Reggeon $\mathcal{A}_\pm$ couples to the incomming particle with large k_- and $\mathcal{A}'$ to the one with large k_+ looks more complicated. Similar to the $\mathcal{A}'-\mathcal{A}'$ case in Sec. V C besides $\langle \mathcal{L}_{\rm tot}^{(4)} \mathcal{L}_1^{(3-)} \rangle$, more contributions have to be evaluated explicitly. The result $\mathcal{L}_p|_{\mathcal{A}_- - \mathcal{A}'}$ can be obtained, however, from the above expression (5.16) by interchanging the labels $+ \leftrightarrow -$ on the Reggeon field and longitudinal derivatives and by replacing $(1/\partial) A_s^{*a} \leftrightarrow -(1/\partial^*) A_s^a$, which is just the P-parity transformation (4.4).

E. Vertices with color-symmetric pre-Reggeons

Analogous to the previous subsection we consider first the case that the non-octet Reggeon $B^{r(\pm)}$ is in the t channel connected to the large k_- scattering particles. It is not difficult to check that there is no gluon production vertex with the Reggeons $B^{r(\pm)}$ in both t channels. The vertex with $B^{r(+)}$ coupling to large k_- can be read off from the corresponding terms in $\langle \mathcal{L}_{\rm tot}^{(4)} \mathcal{L}_1^{(3-)} \rangle$. The relevant projection is Eq. (4.13):

$$\mathcal{L}_{\rm tot}^{(4)}|_D = \frac{g^2}{8} j_D^r \left\{ \left(\frac{\partial^*}{\partial_+} \mathcal{A}_+ D^r \Box \frac{1}{\partial^* \partial_+} \mathcal{A}_+ \right) + i \left(\frac{\partial^*}{\partial_+} \mathcal{A}_+ D^r \frac{1}{\partial^*} \mathcal{A}' \right) + \text{c.c.} \right\} - \frac{g^2}{4} j_D^r j_D^r. \tag{5.17}$$

We consider first the case of the Reggeons $\mathcal{A}_\pm$ or $\mathcal{A}_{(\pm)}$ in the other t-channel; i.e., we contract

$$\mathcal{L}_1^{(3-)}|_{\mathcal{A}_+} = \frac{g}{4} \left(\frac{\partial_-^2}{\partial \partial^*} J_+^a \right) \mathcal{A}_+^a \tag{5.18}$$

and obtain

$$\langle \mathcal{L}_{\rm tot}^{(4)} \mathcal{L}_1^{(3-)} \rangle_{B^{(-)}-\mathcal{A}_+} = \frac{g^3}{32} j_D^r \left\{ 2 \left(\frac{\partial^*}{\partial} A_s^* D^r \frac{1}{\partial^* \partial_+} J_+ \right) + \left(\frac{\partial^*}{\partial} A_s^* D^r \frac{\partial_-}{\partial \partial^{*2}} J_+ \right) \right\} + \text{c.c.} \quad (5.19)$$

We recall the notation for the brackets with D^r,

$$(\mathcal{A}_1 D^r \mathcal{A}_2) B^r = \mathcal{A}_1^a \mathcal{A}_2^b B^r D_{ab}^r , \quad (5.20)$$

and introduce the notation

$$(B \widetilde{D}^a \mathcal{A}_2) \mathcal{A}_1^a = \mathcal{A}_1^a \mathcal{A}_2^b B^r D_{ab}^r \quad (5.21)$$

for the same expression for convenience in order to write

$$\langle \mathcal{L}_{\rm tot}^{(4)} \mathcal{L}_1^{(3-)} \rangle_{B^{(-)}-\mathcal{A}_+} = \frac{g^3}{32} \left\{ -2 \frac{\partial_-}{\partial} \left(j_D \widetilde{D}^a \frac{1}{\partial^*} J_+ \right) - \partial^* \left(j_D \widetilde{D}^a \frac{\partial_-}{\partial \partial^{*2}} J_+ \right) \right\} \frac{1}{\partial} A_s^{*a} \quad (5.22)$$

$$+ \text{c.c.} \quad (5.23)$$

The remaining contributions in Eq. (5.2) lead to the dressing substitution $J_+ \to J_{R_+}$. We obtain the effective production vertices by factorizing $\mathcal{L}_{s\pm}$:

$$\mathcal{L}_p|_{B^{(-)}-\mathcal{A}_+} = \frac{g}{2} \left\{ \frac{\partial_-}{\partial} (B^{(-)} \widetilde{D}^a \partial \mathcal{A}_+) - \frac{1}{2} \partial^* \left(B^{(-)} \widetilde{D}^a \frac{1}{\partial^*} \mathcal{A}_{(+)} \right) \right\} \frac{1}{\partial} A^{a*} + \text{c.c.} \quad (5.24)$$

The vertex $\mathcal{L}_p|_{\mathcal{A}_- - B^{(+)}}$ can be obtained by the substitutions discussed at the end of Sec. V D which are implied by parity symmetry. The calculation directly from Eq. (5.2) takes more terms to evaluate.

Now we consider the case of a pre-Reggeon $\mathcal{A}_s'$ in the second t channel. There is no vertex with $B^{r(-)}$ in one and $\mathcal{A}_2'^{(+)}$ in the other t channel. This is easily seen because $\mathcal{L}_1^{(3-)}$ does involve j_s but not j.

We contract $\mathcal{L}_{\rm tot}^{(4)}|_D$ with

$$\mathcal{L}_1^{(3-)}|_{\mathcal{A}'} = -g j_s^a \mathcal{A}'^a \quad (5.25)$$

and obtain

$$\langle \mathcal{L}_{\rm tot}^{(4)} \mathcal{L}_1^{(3-)} \rangle|_{B-\mathcal{A}'} = \frac{ig^3}{8} \partial^* \left(j_D \widetilde{D}^a \frac{1}{\partial^*} j_s \right) \frac{1}{\partial} A^{*a} + \text{c.c.} \quad (5.26)$$

The remaining contribution in Eq. (5.2) leads to the dressing substitution $j_s \to j_{Rs}$. We obtain the effective production vertex by factorizing $\mathcal{L}_{s\pm}$:

$$\mathcal{L}_p|_{B-\mathcal{A}'} = -\frac{ig}{4} \partial^* \left(B^{(-)} \widetilde{D}^a \frac{1}{\partial^*} A_s'^{(+)} \right) \frac{1}{\partial} A^{*a} + \text{c.c.} \quad (5.27)$$

The vertex $\mathcal{L}_p|_{\mathcal{A}'-B}$ can be obtained by the parity substitution (4.4).

VI. THE HIGH-ENERGY EFFECTIVE ACTION

The effective action of high-energy scattering is obtained from the effective action of quasielastic scattering (4.16) by adding the effective production vertices $\mathcal{L}_p$:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff,scatt} + \mathcal{L}_p . \quad (6.1)$$

The production vertices $\mathcal{L}_p$ have been obtained for all pre-Reggeons appearing in gluodynamics in the vicinity of $j=1$ and $j=0$ in Eqs. (5.7), (5.14), (5.16), (5.27):

$$\mathcal{L}_p = \mathcal{L}_p|_{\mathcal{A}_- - \mathcal{A}_+} + \mathcal{L}_p|_{\mathcal{A}'-\mathcal{A}'} + \mathcal{L}_p|_{\mathcal{A}'-\mathcal{A}_+} + \mathcal{L}_p|_{\mathcal{A}_- - \mathcal{A}'} + \mathcal{L}_p|_{B-\mathcal{A}_+} + \mathcal{L}_p|_{\mathcal{A}_- - B} + \mathcal{L}_p|_{\mathcal{A}'-B} + \mathcal{L}_p|_{B-\mathcal{A}'} . \quad (6.2)$$

The effective action can be written in terms of the complex field $\phi^a = i(1/\partial) A_s^{*a}$, describing the two helicity states of scattering gluons, and the pre-Reggeon fields $\mathcal{A}_\pm, \mathcal{A}_{(\pm)}, \mathcal{A}_s'^{(\pm)}, \mathcal{A}'^{(\pm)}, B^{(\pm)}$.

The parity transformation, interchanging the incoming particles, acts on the terms of the action by interchanging labels $+ \leftrightarrow -$ accompanied by interchanging ϕ and ϕ^* (i.e., inversion of the helicities of scattering gluons). $\mathcal{L}_{\rm kin}$ is symmetric under this transformation. The currents representing the scattering vertices J_-, J, J^*, j_s, j, j_D of large k_- transform into the corresponding currents of vertices with large $k_+, J_{R+}, J_R, J_R^*, j_{Rs}, j_R, j_{RD}$, respectively. This means that $\mathcal{L}_{s+}$ and $\mathcal{L}_{s-}$ transform into each other. The production vertices $\mathcal{L}_p$ are symmetric under the parity transformation. In this way we check that the whole effective action is parity symmetric, as expected.

In the effective action the longitudinal and transverse space-time dimensions are separated to a large extent. Besides of the kinetic term of the scattering fields ϕ, all terms are invariant under separate scale transformations in longitudinal and in transverse directions. All terms are invariant with respect to longitudinal Lorentz boosts and with respect to rotations in the transverse plane.

The notations for the pre-Reggeon fields have been chosen in such a way that they all are invariant with respect to transverse rotations. The rotation acts on the complex numbers x describing the impact parameters as

$$x \to e^{i\alpha} x . \quad (6.3)$$

The (transverse) gluon fields A transform in the same way and the derivatives in the opposite way: $\partial \to e^{-i\alpha} \partial$.

We have written all vertices in $\mathcal{L}_p$ in such a way that no transverse derivatives act on $\phi^a = i(1/\partial) A_s^{*a}$ or ϕ^*, which are invariant under Eq. (6.3). We consider the vertices as transitions of a ''left'' (coupling to large k_- gluons) to a ''right'' (coupling to large k_+ gluons) pre-Reggeon. Consider, for example, the leading pre-Reggeon vertices

$$-ig[(\partial^* \mathcal{A}_- T^a \partial \mathcal{A}_+)\phi^{*a} + (\partial \mathcal{A}_- T^a \partial^* \mathcal{A}_+)\phi^a]. \quad (6.4)$$

In the production term of ϕ^* the state related to the left Reggeon $\mathcal{A}_-$ transforms as $e^{i\alpha}$, i.e., it has transverse angular momentum $n=1$. One can read this term as a transfer of the angular momentum $n=1$ from the ''left'' to the ''right'' pre-Reggeon. In the production vertex of ϕ the angular momentum $n=-1$ is transferred from the left to the right.

We observe that in general non-negative angular momentum $n \geqslant 0$ is transferred from left to right in all production vertices of ϕ^*, whereas in the terms of ϕ we have the transfer of $n \leqslant 0$ only. In the ϕ^* vertices we encounter $n=1$ in the leading $(\mathcal{A}_-, \mathcal{A}_+)$ vertex, $n=0,1,2$ in the other vertices. Notice, in particular, that the transfer of $n=1$ is absent in the nonleading positive-parity pre-Reggeons $\mathcal{A}_{(\pm)}$ and that there is only zero angular momentum transfer in the $(\mathcal{A}_2'^{(-)}, \mathcal{A}_2'^{(+)})$ vertex.

VII. SUMMARY

In this paper we have extended the high-energy effective action to the next-to-leading exchange contributing to $\mathcal{O}(s^0)$ to the amplitude. Staying within the multi-Regge approximation, i.e., assuming the s-channel multiparticle intermediate states obey the conditions (2.6), we improve the approximation in all transformations related to the separation of modes, integration over heavy modes, and extraction of effective vertices keeping all terms suppressed by one power of s compared to the leading ones. We have introduced pre-Reggeon fields describing in the effective action the next-to-leading exchanges. For this the factorizability of the quartic terms for high-energy scattering is essential. We encounter several pre-Reggeons at the level $\mathcal{O}(s^0)$: Some of them carry the gauge group representation of the gluon and some of them other representations. Among the first ones we have the next-to-leading partner of the leading Reggeized gluon carrying positive P parity and insensitive to the helicity of scattering partons. Two pre-Reggeons carry odd P parity and are sensitive to the helicity. The physical difference between the latter is one of the questions to be investigated in more detail. One of them is sensitive to the transverse momenta and the other not. More generally, it has to be investigated whether all these pre-Reggeons Reggeize in the same way as the leading gluon exchange.

The effective vertices describing emission or absorption of a gluon from the exchanged pre-Reggeons is an essential part of our result. Whereas the corresponding effective vertices in the case of the leading gluons and quarks [1,14] were known long before the effective action approach to high-energy scattering was proposed, the effective vertices for the next-to-leading exchanges appear here for the first time. To investigate these vertices more closely will be our next task.

The investigations along these lines contribute to a deeper understanding of the high-energy asymptotics of gauge theories. There is the hope that some of the interesting structure uncovered in perturbation theory is significant in general. On the other hand, there are problems arrising from the present day phenomenology of semihard processes to which the next-to-leading high-energy effective action can be applied.

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